How do we calibrate the calorimeter on orbit?

J. Eric Grove
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mod. Nov 1998 to fix dE/dx IDL routine
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It is important to have an absolute energy calibration of the GLAST calorimeter as it operates on orbit. Radionuclide dopants or alpha flashers can provide several MeV equivalent energy loss and could be used to calibrate the highest gain scale (i.e. lowest energy range). Similarly an actively controlled LED flasher system could provide a programmable energy point, although the mechanical complexity of delivering an LED pulse to each crystal could be high, and there is some concern about the stability of the optical coupling and active feedback mechanism over time.

The high flux of relativistic heavy galactic cosmic rays (GCRs) provides an alternative on-orbit calibrator. Minimum-ionizing C and Fe deposit typically ~800 MeV and ~15 GeV in each CsI bar (the energy deposition is of course linear with pathlength, but reasonable pathlength corrections can be made from reconstructed tracks through the hodoscopic calorimeter). The rate of these GCRs, ~50 Hz, is adequately high to allow calibrations to accumulate in a reasonable time, but not so high that the downlink data volume is stressed.

A significant, unresolved source of uncertainty in deriving an absolute calibration is in the relative scintillation efficiencies of heavy ions and EM showers (see below). A detailed literature search is required here, as well as perhaps a trip to a relativistic heavy ion beam. In any case, the scintillation efficiency does not hinder our generating a relative calibration, from bar to bar.

General scenario

The ACS would be configured to veto events that deposit some fraction of a MIP to several MIPs. Energy depositions of >several MIPS would be flagged. This would flag GCRs from carbon upward in Z, as well as a fraction of Li, Be, and B, although the abundance of LiBeB at the top of the atmosphere is only ~1/4 of the abundance of CNO. The GCRs visible in the GLAST orbit have energies at or above the minimum-ionizing energy and will penetrate the calorimeter (except for highest Z at large angles). For each valid particle, the full GLAST array would be triggered and the data telemetered with gamma-ray science data or in a separate calibration slot. On the ground, trajectories would be derived from the hodoscopic calorimeter itself, or precisely determined from the tracker if possible. After correcting for the derived pathlength in each CsI bar, the dE/dx would be accumulated. An adequate calibration could be derived every day from the ~1000 non-interacting CNO-group nuclei that pass through each CsI bar (see below).

Particle fluxes

I estimated integral and differential fluxes of GCRs in several charge groups at solar minimum for a 28.5 deg inclination orbit using NRL's CREME96 code. The orbit-averaged differential spectrum of C is shown in Appendix A. The spectrum is cut off sharply below ~2 GeV/n by the magnetosphere, has a broad peak with roughly constant flux up to ~6 GeV/n, and above ~10 GeV/n follows the galactic power law with number index 2.7. Thus GLAST is exposed primarily to a minimum-ionizing beam of ~2-6 GeV/n GCRs.

For concreteness in converting the integral fluxes into observed count rates, I required that a GCR pass through both the upper and lower faces of the calorimeter: the geometry factor for this configuration, parallel planes 1.6m on a side and separated by 0.3m, is 8 m^2 sr.

Z range	Flux (m^{-2} sr^{-1} s^{-1})	Rate (s^{-1})
1-28	127	1020
6-28	1.55	12.4

10-28	0.45	3.6
24-28	0.087	0.7

Note that since the geometry factor of the full GLAST instrument is about a factor of 4 higher (i.e. ~30 m^2 sr) and since I am *not* proposing a modification to the on-board trigger to require GCRs to pass through the upper and lower faces, the ***trigger rate of Z>6 GCRs will be about 50 s^{-1}***, a factor of 4 higher than is listed in the table.

The ranges of C and Fe at 2 GeV/n in CsI are 440 g/cm^2 and 110 g/cm^2, respectively. Thus all incident C will penetrate the $10X_0$ calorimeter, as will vertical Fe, while large-angle Fe will stop. (Note that if we remove the criterion that the GCR trajectories pass through the upper and lower faces of the calorimeter, even the C can be stopped in the ~700 horizontal g/cm^2 of CsI). To the extent that particles slow down appreciably in the calorimeter, we can fit the Bragg curve and estimate the total incident energy with some precision.

As an aside, a quick check shows that the calorimeter doesn't make a good mass spectrometer for Fe-group GCRs using the range-energy technique. The range-energy relation can be approximated as a power-law in energy per nucleon, i.e. $R \propto A_p E^\eta$, where A_p is the mass number of the projectile and η ~ 1.2-1.3 or so for our energies and materials. A simple propagation of errors shows that the measurements of E and R must be precise to ~0.1% in order to reduce the contribution from each to 0.25 amu or less. The calorimeter will not achieve this performance. Too bad.

Nuclear interactions

The majority of GCRs suffer nuclear interactions as they pass through the calorimeter. The spallation fragments are rather forward-directed and will likely pass through the same bars that the parent passes. Whether charge-changing interactions can be identified from the energy loss profile depends on the parent species. For penetrating Fe, the majority of interactions, which involve p-stripping or α-stripping, may very well be undetectable given the relatively small change in dE/dx (26^2 to 25^2 MIPs in the former case) and the uncertainty in energy calibration from bar to bar. In contrast for stopping Fe, interactions are quite easily identifiable from the MIP or larger signals from the spallation fragments in the bars following the stopping point of the primary particle. For C primaries, all of which penetrate, the change in dE/dx is rather substantial (30% or greater), and it should be a relatively simple matter to weed out interactions.

I calculated nuclear interaction lengths from the fit to the "Bradt-Peters charge-changing cross sections with overlap" given in Westfall et al. (1979):

$$\sigma_{pt} = \pi r^2 (A_p^{1/3} + A_t^{1/3} - b)^2$$

where A_p is the mass number of the projectile, A_t the mass number of the target, r = 1.47 fm, and b = 1.12 for targets other than hydrogen. The mean free path is then

$$\lambda_{pt} = \Sigma\, m_t / \ \Sigma\, \sigma_{pt}$$

or for the GCRs in the calorimeter (see Appendix B), $\lambda_{N,CsI}$ = 85.6 g/cm^2 and $\lambda_{Fe,CsI}$ = 58.2 g/cm^2. Assuming that a typical GCR traverses the $10X_0$ calorimeter at ~30 deg (i.e. a total of 96.5 g/cm^2), ~30% of the CNO group and ~20% of the Fe group survive without interacting.

Rate of non-interacting CNO group through top face and out bottom	$= 8 \times (1.55\text{-}0.45) \times 0.3$	$= 2.5\ s^{-1}$
Rate of non-interacting Fe group …	$= 8 \times (0.087) \times 0.2$	$= 0.15\ s^{-1}$

Since I have required that GCRs pass through the top and bottom surfaces, each particle will pass through one of the 250 CsI bars in each layer. Thus I estimate that there will be ***~900 non-interacting CNO-group and ~50 non-interacting Fe-group nuclei in each CsI bar every day.***

Scintillation efficiency

It would be ideal if our detectors were perfectly linear, that is, if the scintillation light yield were strictly proportional to the ionization energy loss. Note, however, that the light output of CsI(Tl) is not proportional to dE/dx for heavy nuclei, but exhibits a saturation so that the light output per unit energy loss dL/dE decreases slowly with increasing dE/dx. The uncertainty in determining dL/dE translates directly into an uncertainty in deriving an absolute energy calibration from heavy CRs on orbit. Electromagnetic showers presumably suffer different saturation effects.

A detailed literature search is required here to determine how well dL/dE is known for relativistic heavies in CsI(Tl). My own experience is with 500-2000 MeV/n heavies in NaI(Tl), where at these velocities dL/dE is primarily a function of dE/dx rather than the charge of the beam (i.e. Ne and Fe with the same dE/dx [and thus very different energy per nucleon] have similar dL/dE). This saturation is least important near the minimum ionizing energy for each species (in NaI(Tl) it amounts to ~10% correction), although near the end of range the efficiency can drop to ~1/3 or less.

Calibration uncertainty

We can make an educated guess as to the precision of the energy calibration derived with this method. It depends on both the intrinsic energy resolution and the light collection.

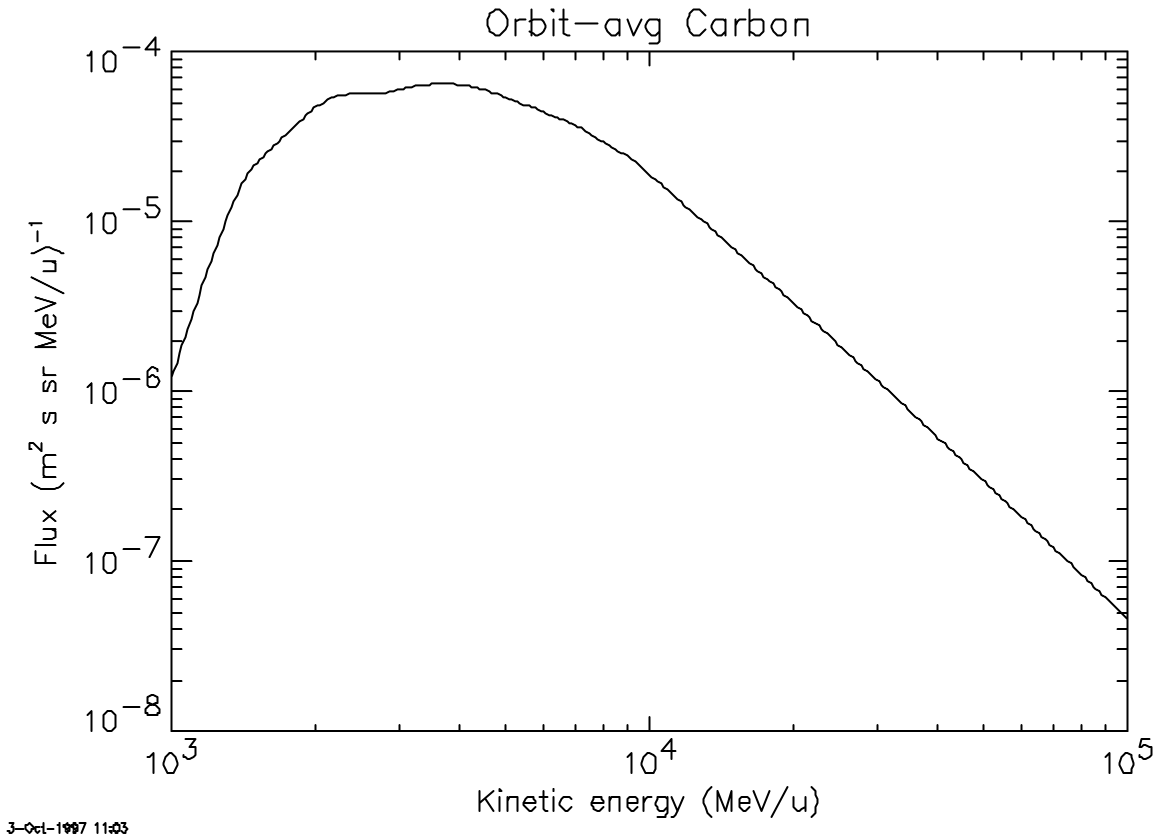
The energy deposited in a crystal will vary from particle to particle of a given Z for the following reasons.
1. *Differences in initial energy*. A simple dE/dx calculator (see Appendix C) shows that ΔE varies ~10% between 2 GeV/n and 6 GeV/n, with ΔE slowly increasing with energy due to the relativistic rise term.
2. *Differences in pathlength*. The energy loss is proportional to pathlength, but GCR tracks should be reconstructed to ~ few degrees with the hodoscopic calorimeter alone and substantially better to the extent that they pass through the tracker. The pathlength should easily be correctable to ~ few % or better.
3. *Landau fluctuations*. Ionization energy loss is a statistical process. For absorbers of moderate thickness, the fluctuations are essentially Gaussian, and the rms variation σ_L can be estimated (see Appendix C) to be ~2-5% for CNO-group at 2-6 GeV/n. Convolving Gaussians with mean=ΔE and rms=σ_L with the CREME96 spectrum gives an asymmetric energy loss spectrum with FWHM ~11% (which I will pretend corresponds to an rms of ~5%). The asymmetry arises from the correlation between E, ΔE, and σ_L (i.e. smaller E has smaller ΔE and σ_L), and the long tail to large energy losses arises from the GCR spectrum.
4. *Nuclear interactions and accompanying fragments*. Interacting C nuclei should be relatively easy to identify and remove from the beam. Unidentified fragments would contribute a low-energy tail, and I've ignored their contribution.

Furthermore, the scintillation light yield will vary for the following reasons.
1. *The position dependence of light-collection efficiency*. The scintillation light yield varies something like 30% from end to end in the CsI bars and should be mapped before flight. Particle trajectories will allow corrections to observed light yield. Unmapped, residual variations should be no larger than 10%.
2. *The energy dependence of dL/dE*. To the extent that the calibration is derived from CNO rather than Fe-group, all the particles are minimum-ionizing and penetrating, and this effect should be negligible.

Adding all of these sources of uncertainty in quadrature, I estimate that the rms variation in measured energy loss in each bar will be <20%. The energy scale is then defined by the mean of the measured energy loss distribution and the expected value. Thus with ~1000 CNO per CsI bar per day, an ***energy calibration with a statistical precision of ~1% should be achievable each day***.

Appendix A. GCR Carbon Spectrum

From CREME96, the orbit-averaged differential flux of GCR carbon transmitted through the magnetosphere at solar minimum is shown in the following figure. Note the broad, flat peak in the spectrum, such that the beam is predominantly ~2-6 GeV/nuc particles.



Appendix B: Nuclear interactions

IDL code fragment for calculating the mean-free-paths for charge-changing interactions for various cosmic ray beams in CsI.

```
; cross-section for charge-changing interactions (Bradt-Peters)
r0 = 1.35e-13                       ; cm
b = 0.83                            ; Westfall's overlap factor
; target materials are Si, I, Cs, Pb
; beam particles are He, C, N, O, Ne, Si, Fe
Atarg = [28,127,133,207.] # replicate(1.,7)
Ainc = replicate(1.,4) # [4,12,14,16,20,28,56.]
sigma = !pi * r0^2 * (Atarg^0.333 + Ainc^0.333 - b)^2

; mean free path (g/cm2)
Navo = 6.023e23                     ; atoms per g-atom
mfp_Si = reform((Atarg(0,*)/Navo) / sigma(0,*))
mfp_CsI = total(Atarg([1,2],*)/Navo,1) / total(sigma([1,2],*),1)
mfp_Pb = reform((Atarg(3,*)/Navo) / sigma(3,*))

print, 'Mean free path for charge-changing interactions'
print, '         ', ['He','C ','N ','O ','Ne','Si','Fe'], form='(a,7a7)'
print, 'on Si:    ', mfp_Si,  'g/cm2', format='(a,7f7.1,a10)'
print, 'on CsI:   ', mfp_CsI, 'g/cm2', format='(a,7f7.1,a10)'
print, 'on Pb:    ', mfp_Pb,  'g/cm2', format='(a,7f7.1,a10)'

;Mean free path for charge-changing interactions
;             He      C      N      O     Ne     Si     Fe
;on Si:      56.5   40.3   38.2   36.4   33.6   29.6   22.4     g/cm2
;on CsI:    111.5   88.8   85.6   82.9   78.3   71.5   58.2     g/cm2
;on Pb:     135.3  110.7  107.2  104.1   99.0   91.3   75.9     g/cm2
```

Appendix C: Energy loss and fluctuations

IDL code to calculate dE/dx and Landau fluctuations is given below.

Energy loss

As a particle of charge Z and mass number A passes through matter, it loses energy predominantly through collisions with atomic electrons. The specific ionization energy loss for a heavy ion can be approximated with the following relation, which includes a relativistic term, but ignores corrections for the atomic shell-effect (e.g. Barkas and Berger 1964), the relativistic density effect (Fermi 1939, 1940), Mott scattering, the non-relativistic Bloch effect (1933), and the relativistic Bloch effect (e.g. Ahlen 1982).

$$\frac{dE}{dx} = 4p\, r_0^2\ m_e c^2 N_A \frac{Z_T}{A_T} \frac{Z_{eff}^2}{ß^2} \left[\ln\left(\frac{2m_e c^2 ß^2 ?^2}{I_{adj}} \right) - ß^2 \right]$$

$$\frac{dE}{dx} = 0.301\ m_e c^2 \frac{Z_T}{A_T} \frac{Z_{eff}^2}{ß^2} \left[\ln\left(\frac{2m_e c^2 ß^2 ?^2}{I_{adj}} \right) - ß^2 \right] \frac{MeV}{g/cm^2}$$

where r_0^2 is the classical electron radius; N_A is Avogadro's number; ρ_T, Z_T, A_T, and I_{adj} are the density, charge, mass, and logarithmic mean ionization potential of the target material; and $Z_{eff} = Z\,[1 - \exp(-130\beta Z^{-2/3})]$ is the effective charge of the projectile with velocity β and Lorentz factor γ.

Landau fluctuations

For moderate and high-Z particle beams, the distribution of energy losses in a thick target is approximately Gaussian and, under the assumption of negligible slowing-down of the beam in the target, has variance given by (Rossi 1952)

$$\sigma_L^2 = 0.301\ m_e c^2\ Z^2\ \frac{Z_T}{A_T} \left(\frac{1}{ß^2} - \frac{1}{2} \right) \Delta x\, E'_{max} \quad MeV^2$$

where $E'_{max} = 2\, m_e c^2\, \beta^2\, \gamma^2$ is the maximum energy that can be imparted to a knock-on electron, and Z_T and A_T are the charge and mass number of the target material of thickness Δx g/cm^2.

```
; beam has charge Z, velocity beta, Lorentz factor gam
; target has charge Zt, mass At, thickness dx (g/cm2)

Z = 6 & A = 12
Ebeam = [2e3,4e3,8e3,10e3,20e3]     ; beam kinetic energy (MeV/nuc)
dx = 2.5 * 4.51 / cosd(30)    ; 2.5 cm of CsI (4.51 g/cm2) at 30deg
Zt = 54.                      ; avg of Cs and I
At = 130.                     ; avg of Cs and I
rhot = 4.51                   ; density of CsI (g/cm2)
Iadj = 535.e-6                ; avg of Cs and I (MeV)
r0 = 2.818e-13                ; classical electron radius (cm)
Navo = 6.023e23               ; atoms per g-atom
mec2 = 0.511                  ; electron rest mass (MeV)
muc2 = 931.5                  ; amu rest mass (MeV)

gam = 1 + Ebeam/muc2
beta = sqrt(1 - 1/gam^2)
```

```
; max energy that can be imparted to KO electron
Emax = 2 * mec2 * beta^2 * gam^2

; for thick enough slabs, if there is negligible slowing down, energy
;   loss distribution is gaussian with rms =
rms = sqrt(0.301 * mec2 * Z^2 * (Zt/At) * (1/beta^2 - 0.5) * dx * Emax)

; simple dE/dx form, leaving out corrections beyond relativistic rise
Zeff = Z * (1 - exp(-130*beta*Z^(-2/3.)))
dedx = (4*!pi * r0^2 * mec2 * Navo * (Zt/At) * Zeff^2) / beta^2
dedx = dedx * (alog(2*mec2*beta^2*gam^2/Iadj) - beta^2)

; energy lost in bar
dE = dedx * dx

; fractional energy resolution
frac = rms / dE

print, 'Carbon beam on CsI bars'
print, Ebeam, format='(5f10.0)'
print, frac, format='(5f10.4)'

; find the residual energy after traversing HodoCal
Ebeam = Ebeam - dE/A
.run
for lay=1,7 do begin
    gam = 1 + Ebeam/muc2
    beta = sqrt(1 - 1/gam^2)
    Emax = 2 * mec2 * beta^2 * gam^2
    rms = sqrt(0.301*mec2* Z^2 * (Zt/At) * (1/beta^2 - 0.5) * dx * Emax)
    dedx = (4*!pi*r0^2 * mec2 * Navo * (Zt/At) * Zeff^2) / beta^2
    dedx = dedx * (alog(2*mec2*beta^2*gam^2/Iadj) - beta^2)
    dE = dedx * dx
    Ebeam = Ebeam - dE/A
    print, 'At end of layer ', strtrim(lay+1,2)
    for ee=0,n_elements(ebeam)-1 do $
      print,'    ',Ebeam(ee),dE(ee),rms(ee),rms(ee)/dE(ee),$
      form='(a,3f10.1,f10.4)'
endfor
end
```

This is the end.